A RELIABLE INTERNAL RATE OF RETURN

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The Usual Internal Rate of Return (IRR)

The internal rate of return (IRR) is a widely used measure of financial evaluation. It is the discount rate for which a project’s benefits exactly equal its costs; in other words, it is the rate at which the project’s net present value is zero. The IRR is shown diagrammatically in Figure 1 below and expressed formally in Equation 1.

The IRR is that $r$ which satisfies equation 1 for a given $T, B_i$ and $C_0$.

$$\sum_{t=1}^{T} \frac{B_i}{(1 + r)^t} - C_0 = 0$$

(1)

It is well known that the IRR suffers from two defects not found in the NPV criterion. First, choosing among projects based solely on the IRR may cause a project with lower NPV (and therefore lower absolute benefits) to be selected. As demonstrated in Figure 2 below, this occurs when the discount or trigger rate is below the rate at which the present values of the two projects

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are equal. In Figure 2, Project A is superior to Project B at any rate below $r^*$.\(^2\) In Figure 3, each project has the same IRR but Project A is superior to Project B at any discount rate.

![Figure 2 The IRR Does Not Necessarily Tell Which Project is Better](image)

\(^2\) Difficulties in finding the IRR are explored in [http://search.cpan.org/~erwan/Finance-Math-IRR-0.10/lib/Finance/Math/IRR.pm](http://search.cpan.org/~erwan/Finance-Math-IRR-0.10/lib/Finance/Math/IRR.pm). The Modified internal rate of return (MIRR) has been developed to address the problems with IRR: rather than assuming equal rates of growth for interim cash flows, the MIRR uses the cost of capital as the rate of growth of interim cash flows; in addition, while the IRR may yield various answers, the MIRR yields only one (see Investopedia [http://www.investopedia.com/terms/m/mirr.asp](http://www.investopedia.com/terms/m/mirr.asp)), which defines the MIRR as the cost of capital when the NPV discounted by the cost of capital is positive.

Because the MIRR yields lower interest rates than the IRR, it is possible to envisage a situation where using the IRR results in a positive NPV (good project), and using the MIRR results in a negative NPV (bad project). Therefore, using the MIRR would lead to the better financial decision.

The problem with MIRR is that it is not useful when comparing two projects. Two projects can have a positive NPV when discounted at the MIRR but one can have still have a larger NPV (the logic depicted in Figures 2 and 3 still applies).
Another situation in which the IRR would not be useful in choosing among projects is when the IRRs of projects intersect more than once. This would be the case when the rates of benefits and rates of costs fluctuate over the duration of the project. Such a case could look like Figure 4 below, where Project A dominates Project B for some time horizons, but is dominated by Project B for others.
The second defect of the IRR criterion is that a project with a mix of positive and negative cash flows over time does not have a unique IRR. Indeed, as Figure 4 illustrates, there may be up to as many possible IRRs for a project as there are sign changes in the cash flows of the project. This results in two problems: there may be several IRRs for a project, and all of these may be incorrect. Consider for example a project with the cash flows described in Table 1 below:

Table 1 Cash Flows For IRR That Yield Two Answers

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-1,000</td>
</tr>
<tr>
<td>Year 1</td>
<td>9,000</td>
</tr>
<tr>
<td>Year 2</td>
<td>-9,500</td>
</tr>
<tr>
<td>IRR</td>
<td>22.13%</td>
</tr>
<tr>
<td>NPV</td>
<td>($1,216.80)</td>
</tr>
</tbody>
</table>

In this case, here are two positive IRRs, and both are large; yet, the NPV for the project using a 3% discount rate shows that the project is not financially desirable. The reason the IRR can give multiple results is that the IRR assumes that interim cash flows all increase at the same rate: that of the IRR itself. Thus, the IRR of 22.13% assumes that cash flows increase by 22.13% every year until the end of the second year; the IRR of 677.9% assumes that cash flows increase at a rate of 667.9% for every year in that period.

In sum, the IRR can lead to an incorrect project choice both because of its implicit assumption of an unrealistic rate of cash flow growth and because the project with the higher IRR may not necessarily have the higher NPV (and thus yield the greater increase in wealth).

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4 Note that if the discount rate were greater than the 22.13% IRR, the NPV would be positive because the $9,500 loss in the second year would be significantly discounted.
The Reliable IRRr

Reliability

The deficiencies discussed in the previous section can be overcome by making explicit a) the opportunity cost of funds (the discount rate) and b) the budget of the projects being compared. Indeed, a more elegant and informative method for finding an “internal rate of return” is to find that rate at which the initial investment would have to grow to yield the future value of the remaining cash flows – in other words, to find the rate of increase in real wealth generated by a project.

If we take the reliable internal rate of return (IRRr) to be the measure of the growth of the original investment \( C_0 \) over the lifetime of the longest project \( T \), and we understand \( F \) to be the future value of the project at the end of that lifespan, then:

\[
F = C_0(1 + \text{IRRr})^T
\]  
(3)

In calculating the future value of the project’s cash flows, we assume that these cash flows can be invested at the opportunity cost of capital (the discount rate \( r \)). As such, the future value of the project’s cash flows \( B_t \) can be expressed as:

\[
F = \sum_{t=1}^{T} B_t(1 + r)^{T-t}
\]  
(4)

Since both equations 3 and 4 express the future value of the project, they can be set equal to one another to find an equation for the IRRr: We ask at what rate does the initial investment grow to yield the future account. Thus:

\[
C_0(1 + \text{IRRr})^T = \sum_{t=1}^{T} B_t(1 + r)^{T-t}
\]  
(5)

Equation 5 can then be rearranged as follows:

\[
\text{IRRr} = \left( \frac{\sum_{t=1}^{T} B_t(1 + r)^{T-t}}{C_0} \right)^{1/T} - 1
\]  
(6)

The main difference between the IRR and the IRRr – the elimination of the unrealistic assumption that interim cash flows can be reinvested at the IRR rate – makes the IRRr accurate in cases where the IRR fails. The IRRr has the additional advantage of always yielding a single answer, regardless of whether project cash flows are positive or negative over its lifespan.
**IRRr and Project Comparison**

The IRRr can reliably be used to compare different projects. Consider, for instance, the following sets of cash flows:

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-100</td>
<td>-100</td>
</tr>
<tr>
<td>Year 1</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>Year 2</td>
<td>60</td>
<td>44</td>
</tr>
<tr>
<td>NPV</td>
<td>14.81</td>
<td>14.29</td>
</tr>
<tr>
<td>IRR</td>
<td>13%</td>
<td>14%</td>
</tr>
<tr>
<td>IRRr</td>
<td>10.36%</td>
<td>10.11%</td>
</tr>
</tbody>
</table>

Table 2 Cash Flows for Two Projects

The IRRr gives the correct answer, while the IRR does not. The IRRr agrees with the NPV, as its rate is greater than the opportunity cost rate of 3%. Both projects are financially viable and of the two A is better as having the higher rate.

As is apparent from equation 6, in order to reliably compare the IRRr of two different projects, these projects must be given 1) the same initial cost basis (that is, the \( C_0 \) for each project must be identical); and 2) equal life spans (i.e. equal \( T \)). Each of these conditions are easy to fulfill and will be discussed in turn below.

**Project Scale**

Two projects with different initial investments can be made equal by taking the largest investment as the “total budget”, and ensuring that any project not exhausting the total budget invests the remainder at the opportunity cost of money. For example, consider the following projects:

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-100</td>
<td>-92</td>
</tr>
<tr>
<td>Year 1</td>
<td>60</td>
<td>75</td>
</tr>
<tr>
<td>Year 2</td>
<td>60</td>
<td>35</td>
</tr>
</tbody>
</table>

Table 3 Projects of Different Scale

Project B does not use the entire budget of $100. We reformulate this accounting for the total budget of $100. This gives:
Table 4  Setup for Projects of Different Scale

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Project A</th>
<th>Project B</th>
<th>Remainder of B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-100</td>
<td>-92</td>
<td>-8</td>
</tr>
<tr>
<td>Year 1</td>
<td>60</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td>60</td>
<td>35</td>
<td>10.609</td>
</tr>
</tbody>
</table>

The two projects can now be compared:

Table 5 Comparing Projects of Different Scale

<table>
<thead>
<tr>
<th></th>
<th>Project A</th>
<th>Project B</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-100</td>
<td>-100</td>
<td></td>
</tr>
<tr>
<td>Year 1</td>
<td>60</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>Year 2</td>
<td>60</td>
<td>45.609</td>
<td></td>
</tr>
<tr>
<td>NPV</td>
<td>$14.81</td>
<td>$13.81</td>
<td></td>
</tr>
<tr>
<td>B/C</td>
<td>1.15</td>
<td>1.14</td>
<td></td>
</tr>
<tr>
<td>IRRr</td>
<td>10.4%</td>
<td>9.98%</td>
<td></td>
</tr>
</tbody>
</table>

Once the scale of the projects is made equal, the NPV, the IRRr and the B/C all give the same decision answer: Project A is better than Project B, and both are worthwhile.

Suppose, however, that we did not account for the remaining budget for project B and attempted to compare the two projects without any adjustment. We will label the incorrect B/C ratios as B/Cx and the incorrect IRRrs as IRRrx. The incorrect answers would then be:

Table 6 Financial Indicators Without Scale Adjustments

<table>
<thead>
<tr>
<th>Financial Indicator</th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>14.81</td>
<td>13.81</td>
</tr>
<tr>
<td>B/Cx</td>
<td>1.148</td>
<td>1.150</td>
</tr>
<tr>
<td>IRRrx</td>
<td>10.4%</td>
<td>14.6%</td>
</tr>
</tbody>
</table>

While the NPV result remains the same, the IRRrx and B/Cx now yield the opposite and incorrect result.
This example demonstrates that, in order to obtain the correct result using the IRRr and B/C when comparing projects with different initial investments, the entire available budget must be accounted for. Using the method explained here will always yield an IRRr consistent with the NPV result as long as the discount rate used for the NPV calculation is also employed as the opportunity cost of funds in the IRRr calculation.  

Project Lifespan

The second condition for comparing projects using the IRRr is to make the projects of equal duration. The easiest way to achieve this is to use the lifespan of the longest project. Consider the two projects below:

Table 7 Evaluation Criterion when Projects are of Different Duration

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 0</td>
<td>-100</td>
<td>-100</td>
</tr>
<tr>
<td>Year 1</td>
<td>60</td>
<td>45</td>
</tr>
<tr>
<td>Year 2</td>
<td>60</td>
<td>45</td>
</tr>
<tr>
<td>Year 3</td>
<td>-</td>
<td>45</td>
</tr>
</tbody>
</table>

To find the IRRr, we find the future value of benefits for both projects at the end of Year 3. We use an opportunity cost of capital of 3%. This gives the following IRRrs.

Table 8 IRR Results: Projects of Different Duration

<table>
<thead>
<tr>
<th></th>
<th>Project A</th>
<th>Project B</th>
</tr>
</thead>
<tbody>
<tr>
<td>IRRr</td>
<td>7.8%</td>
<td>11.5%</td>
</tr>
</tbody>
</table>

The reader can check the accuracy of these results by calculating the NPVs for the projects.

Summary

This short article has shown that an alternative to the IRR, the proposed reliable IRRr, can be used to provide a more reliable and more successful estimate of project returns or comparisons of project returns. In order to find the IRRr of a project or set of projects, the steps to follow are listed below:

(1) Find the future value of project cash flows ($F$) using the best estimate of the rate of growth of benefits (the opportunity cost of capital), making equal the scale and time period of the

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5 Refer to the Appendix for a more detailed discussion of this point.
two projects (i.e. including the future value of the remainder of the budget on the ending date of the longest project);

(2) Divide by the same initial investment (the total budget, $C_0$);

(3) Take the $T^{th}$ root of the result ($T$ being the longest project duration); and

(4) Subtract 1.
**APPENDIX**

**Proof One: The IRRr Will Give the Same Decision Answer as the NPV**

Setup: Prove that if \( NPV > 0 \) then \( IRR > r \), where \( r \) is the opportunity cost of funds / trigger rate / discount rate.

\[
NPV = \sum_{i=1}^{T} \frac{B_i}{(1 + r)^t} - C_0 > 0 \quad \text{by definition}
\]

(note that negative future cash flows are counted as negative benefits for simplicity)

Then, multiplying by \((1 + r)^T\) on both sides yields:

\[
\sum_{i=1}^{T} \frac{B_i}{(1 + r)^t} (1 + r)^T > C_0 \cdot (1 + r)^T
\]

Diving by \(C_0\) and taking the \(T\) root of both sides we have:

\[
\left( \frac{\sum_{i=1}^{T} B_i \cdot (1 + r)^{T-t}}{C_0} \right)^{1/T} - 1 > r
\]

The expression on the LHS is the IRRr, thus:

\( IRR > r \) where \( r \) is the opportunity cost of money, or the trigger rate for the project. If the NPV is negative, the solution to the IRRr is an imaginary number which indicates that the project is not worthwhile.

**Proof 2: The Need to Account for the Remainder**

Setup: If \( C_{0A} > C_{0B} \), then the B/Cx or the IRRrx may give the wrong answer unless the remainder of the budget is accounted for.

Consider the simple case in which there are two alternative projects such that \( NPV_A = NPV_B \) but project B has a smaller initial cost. Clearly, project B will have the higher B/C ratio as its ratio is divided by a smaller C. Now imagine that \( NPV_A = NPV_B + \varepsilon \) where \( \varepsilon \) is very small. Since \( \varepsilon \) can be made vanishingly small, Project B’s NPV can still be less than that for A but its B/C ratio will be larger, giving the incorrect answer.

**Proof 3: The Need to Use the Future Value of the Longest Project**

Setup: When comparing two projects of different duration, the IRRr will yield the incorrect answer unless their duration is made to be identical.

Consider two projects with equal NPVs: Project B has a longer duration. In calculating the future value based on each project’s time period, we will multiply its present value by a number proportional to its duration. Since in using the IRRr the budgets are equal, project B will have a
higher IRRr than A, contrary to what is shown by the NPV. The proof that using the longest lived budget is essentially the same as Proof 1. Simply replace $T$ by $T^*$, where $T^*$ is the duration of the longest project.